A Quantitative Comparison of Soil Moisture Inversion Algorithms

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Abstract-**This paper compares the performance of four bare surface radar soil moisture inversion algorithms in the presence of measurement errors. The particular errors considered include calibration errors, system thermal noise, local topography and vegetation cover.**

I. INTRODUCTION

Several algorithms have been proposed in the literature to invert radar measurements to infer surface soil moisture [1-3]. In all cases, multiple radar channels, usually multiple polarizations, are required to separate the effects of surface roughness and surface dielectric constant in order to extract information about the surface moisture. While these algorithms have been demonstrated to yield accuracies on the order of 4% when inferring surface soil moisture, there is no single comprehensive study that quantifies the differences between these algorithms when applied to imaging radar data, especially in the presence of measurement errors. In one study [4], for example, two algorithms were applied to the same data set, and results were reported on the number of pixels for which the algorithms failed to produce reasonable results. No explanation for the failure of the algorithms to produce results was offered, however.

Also missing in the literature, is the sensitivity of these inversion algorithms to calibration errors, and more importantly, how the presence of vegetation and local topography will affect the inversion results. The only paper that describes the performance of the algorithm in the presence of calibration errors and vegetation is that of Dubois *et al.* [2].

In this paper we critically examine the sensitivity of four algorithms proposed in the literature to various measurement errors. Space does not permit a full description of the results in this short paper; instead, we shall concentrate here on describing the inversion algorithms, and the theoretical framework used for analyzing the effects of the errors. The detailed results will be provided in the presentation at the conference.

II. RADAR INVERSION ALGORITHMS

As mentioned earlier, several algorithms have been proposed to invert measured radar signals to infer soil moisture. We shall now first briefly discuss several of these models before comparing their performance in the presence of the previously mentioned error sources.

A. First-Order Small Perturbation Model

The use of the first-order small perturbation model to describe scattering from slightly rough surfaces dates back to Rice [5]. Using the first-order small perturbation model, one can show that the ratio of the two co-polarized radar cross-sections in the linear basis is independent of the surface roughness, and is given by

$$\sigma_{hh} / \sigma_{vv} = \left| \alpha_{hh} / \alpha_{vv} \right|^2 \tag{1}$$

where

$$\alpha_{hh} = \frac{(\varepsilon - 1)}{(\cos\theta + \sqrt{\varepsilon - \sin^2\theta})^2} \tag{2}$$

and

$$\alpha_{vv} = \frac{(\varepsilon - 1)\left[(\varepsilon - 1)\sin^2\theta + \varepsilon\right]}{(\varepsilon\cos\theta + \sqrt{\varepsilon - \sin^2\theta})^2} \tag{3}$$

Here θ is the local angle of incidence at which the radar waves impinge on the surface and ε is the complex dielectric constant, or relative permittivity, of the soil, which is a function of the soil moisture. Dry soil surfaces have dielectric constants on the order of 2-3, while water has a dielectric constant of approximately 80 at microwave frequencies.

It is possible to rewrite the ratio in (1) as a fourth order polynomial in ε. To invert the radar measurements for surface dielectric constant, one then has to find the roots of this fourth order polynomial. In practice we found it easier and computationally more efficient to simply invert (1) using a look-up table approach.

It should be pointed out that the small perturbation model is only applicable to surfaces that are smooth. This usually means that the surface slopes are small, and that the surface r.m.s. height is small compared to the radar wavelength. While this model predicts the ratio of the co-polarized radar cross-sections to be independent of the surface roughness, the contrary is observed in experimental data. Extending the perturbation model to include second-order terms, one finds that this ratio indeed is affected by the surface roughness, and that the effect of the roughness is to increase this ratio, *i.e.* to make the *vv* cross-section closer to that at *hh* polarization. The net effect of this is that a first-order small perturbation inversion will tend to underestimate the surface dielectric constant in the presence of significant roughness, as we will see in the next section.

B. Algorithm Proposed by Oh et al. (1992)

Based on the scattering behavior in limiting cases and experimental data, Oh *et al.* [1] have developed an empirical model in terms of the r.m.s. surface height, the wave number and the relative dielectric constant. The key to this approach is the co-polarization ratio p and cross-polarization ratio q, which are given explicitly in terms of the roughness and the soil dielectric constant. The parameters p and q from the Oh's algorithm are derived using an empirical fit to the data collected by their truck-mounted scatterometer system over bare soils of different roughness and moisture conditions.

To invert these expressions for the soil dielectric constant, we note that one can rewrite the expressions provided by Oh *et al* [1] in terms of only the surface dielectric constant as

$$\sqrt{p} = 1-\left(\frac{2\theta}{\pi}\right)^{1/3\Gamma_0}\left(1-\frac{q}{0.23\sqrt{\Gamma_0}}\right) \tag{4}$$

We found that the most efficient way to solve this expression for the dielectric constant is to use a look-up table inversion.

C. Algorithm Proposed by Dubois et al. (1995)

Dubois *et al.* [2] developed an empirical model that only requires measurements of the co-polarized radar cross-sections at a frequency between 1.5 and 11 GHz to retrieve both the surface r.m.s. height s and soil dielectric constant from bare soils. They used two sets of ground based scatterometer data collected by Oh *et al.* [1] and by the University of Berne's RASAM system. Rewriting the expressions provided in [2] in the logarithmic domain, one finds two equations in two unknowns. Solving for the dielectric constant yields

$$\begin{aligned}\varepsilon = \{&26.5+14\sigma_{vv}(dB)-11\sigma_{hh}(dB)-255\log_{10}(\cos\theta)\\ &-130\log_{10}(\sin\theta)-21\log_{10}(\lambda)\}/3.36\tan\theta\end{aligned} \tag{5}$$

Note that the wavelength used in these expressions must be in *centimeters*. According to Dubois *et al.* [2], their algorithms is applicable to surfaces with $ks < 3.0$, and $30^0 \le \theta \le 70^0$.

D. Algorithm Proposed by Shi et al. (1997)

A concern about the empirical approaches described so far is that these models do not take into account the shape of the surface power spectrum which is related to the surface roughness correlation function and correlation length. This is not consistent with theoretical surface backscattering model predictions, *i.e.* the backscattering coefficients are sensitive not only to soil moisture and surface r.m.s. height s, but also to the shape of the surface roughness power spectrum. In addition, any empirical model developed from a limited number of observations might give site-specific results because of the nonlinear response of backscattering to the soil moisture and surface roughness parameters. This drawback may be reduced by using data from many different sites as done by Dubois *et al.* [2], but it is very difficult to acquire data experimentally that would cover all possible types of surfaces and include the entire range of expected roughness and moisture conditions.

Progress in theoretical modeling such as the Integral Equation Method (IEM) [6] offers an alternative approach for the retrieval of soil moisture from radar data. Although the IEM model is valid for a wider range of surface roughness conditions when compared to other earlier theoretical models, the complexity of this model makes its application directly to the radar data to infer soil moisture and roughness parameters rather difficult. Since the number of independent radar measurements are usually limited, Shi *et al.* [3] developed a model by parameterizing IEM model-based numerical simulations for a wide range of surface roughness and soil moisture conditions.

Shi *et al.* (1997) examined many different combinations of polarizations of AIRSAR and SIR-C measurements at L band to evaluate their effectiveness in the estimation of soil moisture and surface roughness. The following equations was used to estimate soil moisture from SIR-C and AIRSAR data [3]:

$$\begin{aligned}&10\log_{10}\left[\frac{|\alpha_{vv}|^2+|\alpha_{hh}|^2}{\sigma^\circ_{vv}+\sigma^\circ_{hh}}\right]\\ &=a_{vh}(\theta)+b_{vh}(\theta)10\log_{10}\left[\frac{|\alpha_{vv}||\alpha_{hh}|}{\sqrt{\sigma^\circ_{vv}\sigma^\circ_{hh}}}\right]\end{aligned} \tag{6}$$

In these expressions,. α_{hh} and α_{vv} are the polarization amplitudes for HH- and VV-polarization as in the small perturbation model, that depends only on the dielectric constant of soil ε and the angle of incidence 6 as shown earlier. The constants $a_{vh}(\theta)$ and $b_{vh}(\theta)$ will not be repeated here to save space, and can be found in [3]. We found that the most efficient way to solve this expression for the dielectric constant is to use a look-up table inversion.

III. Measurement Errors

In this paper we consider separately the effects of the following error sources for the measured radar cross-sections:

A. Calibration Accuracy.

This causes the measured radar cross section to be different from the actual by the calibration error, *i.e.*

$$\textit{Measured } \sigma_{xx}(dB) = \textit{Actual } \sigma_{xx}(dB) \pm \delta\sigma_{xx} \tag{7}$$

We use the subscript xx to explicitly show that the different polarization combinations may have different calibration errors.

B. System thermal noise.

We shall analyze the effects of system thermal noise only in the average sense, and not in the strict statistical sense. In the average, system noise adds an amount equal to the noise power to the measured radar cross section. Therefore, in the presence of system noise, the average radar cross section can be written as

$$\begin{aligned} & Measured\ \sigma_{xx}(dB) = Actual\ \sigma_{xx}(dB) \\ & \quad + 10\log_{10}\left(1 + 1/SNR_{xx}\right) \end{aligned} \tag{8}$$

The signal-to-noise ratio for the given polarization combination is simply the ratio of the actual radar cross-section to the noise power in that channel of the radar system.

C. *The effect of topography.*

This changes the local incidence angle to be different from that assumed in the inversion. Slopes facing the radar system will have local incidence angles less than that assumed for a flat surface, while slopes facing away from the radar will have local incidence angles larger than that assumed for a flat surface. Also, slopes in the along-track direction changes the scattering matrix through a rotation of the coordinate axes.

D. *The presence of vegetation.*

We analyze the effects of vegetation by assuming that a fraction f of the soil in the radar resolution element is completely bare, and a fraction 1-f is uniformly covered with a layer of vegetation. The presence of vegetation has two effects on the measured signal. First, the contribution from the soil is attenuated because of the two-way propagation through the vegetation, and second, some scattered energy from the vegetation itself (and possibly terms involving scattering off the vegetation followed by scattering from the ground) is added to the already attenuated soil term. The combined soil-vegetation measured signal can therefore be written as

$$\begin{aligned} & Measured\ \sigma_{xx}(dB) = Soil\ \sigma_{xx}(dB) \\ & \quad + 10\log_{10}\Big(f + (1-f)e^{-2\tau_{xx}} \\ & \quad + (1-f)Vegetation\ \sigma_{xx} / Soil\ \sigma_{xx}\Big) \end{aligned} \tag{9}$$

In the absence of vegetation, f=1, the optical depth is infinite, and the contribution from the vegetation is zero, so that the term on the right will be zero.

Note that except for the effect of topography, the effects of the other error sources are very similar to that of the calibration error, in that one can replace the calibration error in dB with a different term related to the signal-to-noise ratio or the strength of the vegetation scattering relative to that from the underlying soil.

IV. Example Results

As an example of the results, let us consider the case of the Dubois *et al* algorithm given in (5) with calibration errors. We can rewrite this expression as

$$\begin{aligned} \varepsilon = & \{26.5 + 3\sigma_{vv}(dB) + 11(\sigma_{vv}(dB) - \sigma_{hh}(dB)) \\ & - 255\log_{10}(\cos\theta) \\ & - 130\log_{10}(\sin\theta) - 21\log_{10}(\lambda)\}/3.36\tan\theta \end{aligned} \tag{10}$$

The second term on the right shows that the dielectric constant is proportional to the absolute radar cross-section in dB, while the third term on the right indicates that epsilon is also a function of the ratio of the two copolarized cross-sections. In fact, it is stronger function of the ratio that of the absolute radar cross-section.

Writing the left-hand side of (10) as $\varepsilon + \delta\varepsilon$ in the presence of calibration errors, we find that $\delta\varepsilon$ is linearly proportional to the calibration error. The error in the estimate is more sensitive to calibration errors of the copolarized ratio than in the absolute radar cross section.

V. Conclusions

This paper provides the theoretical framework for analyzing the performance of various soil moisture algorithms in the presence of measurement errors. Detailed results for all algorithms will be presented in the paper at the conference.

VI. Acknowledgment

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VII. References

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